Game Theory Practice

Problem 1
Each of three people announces a number from 0 to 100. The person who is closest to $\frac{2}{3}$ of the average wins \$1. Is there any equilibrium where all players announce the same thing? Are there any other equilibria?

Solution: The only equilibrium is all three people announce 0. The easiest way to see this is that whoever says the highest number is going to lose (or tie) and therefore she can do better by lowering her choice. This is always true unless all three people announce zero. This is an equilibrium because you cannot do any better than announcing 0 if the other two people also announce zero.

Problem 2 (hawk and dove)
Two animals are fighting over the same prey. Each can be Passive or Aggressive. Each prefers to be aggressive if its opponent is passive and passive if its opponent is aggressive. Given its own stance, each animal prefers its opponent to be passive than to be aggressive. Write this as a game and find the Nash equilibria.

Solution: Here is one solution:

	P	A
P	$2, 2$	$1, 4$
A	$4, 1$	$0, 0$

You can use other numbers as long as you keep the order of preferences the same. There are two equilibria: $< P, A >$ and $< A, P >$.

Problem 3
Each of n people chooses whether to contribute a fixed amount toward the provision of a public good. The good is provided if an only if at least k people contribute, where $2 \leq k \leq n$; if it is not provided, contributions are not refunded. Each person ranks outcomes from best to worst as follows:

1. any outcome in which the good is provided and she does not contribute

2. any outcome in which the good is provided and she contributes
3. any outcome in which the good is not provided and she does not contribute
4. any outcome in which the good is not provided and she contributes.

Find this game's Nash equilibrium. (Is there a Nash equilibrium in which more than k people contribute? One in which k people contribute? One in which fewer than k people contribute?)

Solution: There are a number of equilibria for this game. No one contributing is an equilibrium (if no one else contributes then there is no point in you contributing). Also any group of exactly k people contributing is an equilibrium. The people contributing don't want to stop contributing because then the good won't be provided. The people not contributing have no incentive to contribute because there are enough people contributing. If more than k people contribute, then each person contributing has the incentive to not contribute (there are enough other people contributing for the good to be provided), so this is not a Nash equilibrium.

Problem 4

Two candidates, A and B, compete in an election. Of the n citizens, k support candidate A and m (=n-k) support candidate B. Each citizen decides whether to vote, at a cost, for the candidate she supports, or to abstain. A citizen who abstains receives the payoff of 2 if the candidate she supports wins, 1 if this candidate ties for first place, and 0 if this candidate loses. A citizen who votes receives payoffs of 2-c, 1-c, and -c in these three cases.

- For $k = m$, find the set of Nash Equilibrium. (Is the action profile where everyone votes a Nash equilibrium? Is there any Nash equilibrium in which the candidates tie and not everyone votes? Is there any Nash equilibrium in which one of the candidates wins by one vote? Is there any Nash equilibrium in which one of the candidates wins by two or more votes?

  Solution: The only Nash equilibrium is for everyone to vote. If everyone else votes and you do not vote, then your candidate does not win which is worse than if you vote (your candidate ties). In every other scenario, someone has an incentive to deviate. For example, if not everyone votes but the candidates tie, then each person who didn't vote has the incentive to deviate because if they vote then their candidate wins. If not everyone votes and one candidate wins by more than two votes, then anyone who voted for the winning candidate has the incentive to not vote because the candidate has enough votes to win without them. If not everyone votes and one candidate wins by exactly one vote, then anyone who did not vote and supports the losing candidate has the incentive to vote as then their candidate will tie instead of losing.