

Game Theory Practice

Problem 1

Each of three people announces a number from 0 to 100. The person who is closest to $\frac{2}{3}$ of the average wins \$1. Is there any equilibrium where all players announce the same thing? Are there any other equilibria?

Problem 2 (hawk and dove)

Two animals are fighting over the same prey. Each can be Passive or Aggressive. Each prefers to be aggressive if its opponent is passive and passive if its opponent is aggressive. Given its own stance, each animal prefers its opponent to be passive than to be aggressive. Write this as a game and find the Nash equilibria.

Problem 3

Each of n people chooses whether to contribute a fixed amount toward the provision of a public good. The good is provided if and only if at least k people contribute, where $2 \leq k \leq n$; if it is not provided, contributions are not refunded. Each person ranks outcomes from best to worst as follows:

1. any outcome in which the good is provided and she does not contribute
2. any outcome in which the good is provided and she contributes
3. any outcome in which the good is not provided and she does not contribute
4. any outcome in which the good is not provided and she contributes.

Find this game's Nash equilibrium. (Is there a Nash equilibrium in which more than k people contribute? One in which k people contribute? One in which fewer than k people contribute?)

Problem 4

Two candidates, A and B, compete in an election. Of the n citizens, k support candidate A and m ($=n-k$) support candidate B. Each citizen decides whether to vote, at a cost, for the candidate she supports, or to abstain. A citizen who abstains receives the payoff of 2 if the candidate she supports wins, 1 if

this candidate ties for first place, and 0 if this candidate loses. A citizen who votes receives payoffs of $2-c$, $1-c$, and $-c$ in these three cases.

- For $k = m$, find the set of Nash Equilibrium. (Is the action profile where everyone votes a Nash equilibrium? Is there any Nash equilibrium in which the candidates tie and not everyone votes? Is there any Nash equilibrium in which one of the candidates wins by one vote? Is there any Nash equilibrium in which one of the candidates wins by two or more votes?)