ECG 790 - Homework 2 - Due 10/20/11

Question 1:
Consider a sealed-bid second-price auction for a single object, where there are only two allowable bids. The two risk-neutral bidders have valuations which are private information and which are drawn from i.i.d. random variables that are uniformly distributed on the interval $[0, 1]$. After observing her own valuation, each of the two bidders simultaneously and independently submits a bid selected from the two-element set $\{0, 2/3\}$ (i.e., the only allowable bids are 0 and $\frac{2}{3}$). The high bidder wins the object and pays the amount of the losing bid; in the event of a tie, the winner is determined by the toss of a fair coin.

(a) Recall that, in the standard second-price auction game where all bids between 0 and 1 are allowed, a bidder never finds it in her interest to bid above her value. Does this property continue to hold in the current game where there are only two allowable bids? Explain precisely.
(b) Solve for the Bayesian-Nash equilibrium of the second-price auction game where there are only two allowable bids. [Hint: A strategy for this game must fully specify which of the two allowable bids is selected by each bidder, for each possible type.]
(c) Compare the efficiency of the Bayesian-Nash equilibrium of this auction game with the Bayesian-Nash equilibrium of the standard auction game where all bids between 0 and 1 are allowed.

Question 2:
Consider the sealed, simultaneous-bid bargaining model with one-sided incomplete information. The valuations of the seller and the buyer (v_s and v_b, respectively) are i.i.d. random variables that are uniformly distributed on the interval [0,1]. Both the buyer and the seller observe v_s before submitting their bids (only the buyer observes v_b). As usual, assume that the sellers valuation is the cost to produce the good and, in case of no trade, the seller does not need to produce the good.

- Define the strategy for each player.
- Assume that the price is $p = p_b$ (i.e. the trade occurs at the buyers price). Solve for a Bayesian-Nash equilibrium of this game.
- Assume that the price is $p = p_s$. Solve for a BNE of this game.
- Assume that the price is $p = \frac{p_b+p_s}{2}$. Solve for a BNE of this game.
- Compute the probability of trade and the expected payoff of the seller in part d. How does this compare to the linear solution we solved for in class where both the buyer and seller's valuation are private value?

Question 3:
Consider a first-price auction with two bidders, in which the bidders types are i.i.d. random variables uniformly distributed on the interval [0,1]. Each bidder knows her own type but only the distribution of her opponents type, and each bidder is risk-neutral. The bidders simultaneously and independently submit sealed bids, and the high bidder wins the item and pays the amount of her bid. However, unlike the games we considered in class, each bidders valuation from winning the item is given by her type raised to the second power, that is, when bidder is type equals t_i, her valuation is given by t_i^2.
(a) Determine the Bayesian-Nash equilibrium of the first-price auction in which each bidder has just two allowable bids, 0 and $\frac{1}{6}$.
(b) Determine the Bayesian-Nash equilibrium of the first-price auction in which each bidder can bid any amount in the interval [0,1].

Question 4:
The following table shows the demand different agents have for varying quantities of a good. If there are a total of 7 units for sale, what is the outcome of a Vickrey Auction?

	$B1$	$B2$	$B3$	$B4$
1	2	4	1	9
2	5	8	2	12
3	8	12	3	13
4	10	12	4	13
5	11	12	5	13